Method for direct observation of coherent quantum oscillations in a superconducting phase qubit

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Time-domain observations of coherent oscillations between quantum states in mesoscopic superconducting systems were so far restricted to restoring the time-dependent probability distribution from the readout statistics. We propose a new method for *direct* observation of Rabi oscillations in a phase qubit. The external source, typically in GHz range, induces transitions between the qubit levels. The resulting Rabi oscillations of supercurrent in the qubit loop induce the voltage oscillations across the coil of a high quality resonant tank circuit, inductively coupled to the phase qubit. It is the presence of these voltage oscillations in the detected signal which reveals the existence of Rabi oscillations in the qubit. Detailed calculation for zero and non-zero temperature are made for the case of persistent current qubit. According to the estimates for decoherence and relaxation times, the effect can be detected using conventional rf circuitry, with Rabi frequency in MHz range.

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I. INTRODUCTION

As is known the persistent current qubit (phase qubit) is one of the candidates as a key element of a scalable solid state quantum processor.[1,2] The basic dynamic manifestations of a quantum nature of the qubit are macroscopic quantum coherent (MQC) oscillations (Rabi oscillations) between its two basis states, which are differed by the direction of macroscopic current in the qubit loop.

Up till now the Rabi oscillations in time domain[3,4] or as a function of the perturbation power[5,6] have been detected indirectly through the statistics of switching events (e.g. escapes into continuum). In either case the probabilitiy $P(t)$, or $P(E)$, was to be obtained and analyzed to detect the oscillations.

More attractive in the long run is a direct detection of MQC oscillations through a weak continuous measurement of a classical variable, which would implicitly incorporate the statistics of quantum switching events, not destroying in the same time the quantum coherence of the qubit.[7,8,9]

In this paper we describe the approach which allows a direct measuring of MQC oscillations of macroscopic current flowing in a loop of a phase qubit. This qubit variety has the advantage of larger tolerance to external noise, especially to dangerous random background charge fluctuations.[10] To be specific, we will use the example of three-junction small-inductance phase qubit (persistent current qubit[2]) where level anticrossing was already observed.[11]

In our method a resonant tank circuit with known inductance L_T, capacitance C_T and quality factor Q_T is coupled with a target Josephson circuit through the mutual inductance M (Fig. 1). The method was successfully applied to a three-junction qubit in classical regime,[12] when the hysteretic dependence of ground-state energy on the external magnetic flux was reconstructed in accordance to the predictions of Ref. 2.

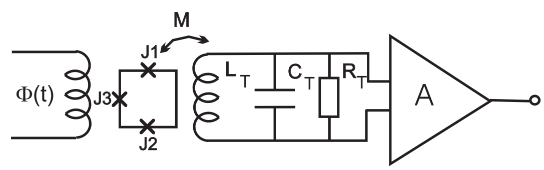

FIG. 1: Phase qubit coupled to a tank circuit.

The phase qubit is biased by external magnetic flux $\Phi(t)$. Assuming small qubit self-inductance, we neglect the shielding current. Therefore the flux through the qubit loop is

$$\Phi(t) = \Phi_x + \Phi_{ac}(t), \tag{1}$$

where Φ_x is a time independent external flux, $\Phi_{ac}(t)$ is a monochromatic high frequency flux from the external source.

If time-dependent external flux is applied to the qubit, the latter will be in a time-dependent superposition of states $|0\rangle$ and $|1\rangle$. If the frequency of external flux is in resonance with the interlevel spacing of the qubit, the average current in the qubit loop will oscillate with the frequency Ω which depends on the amplitude of external flux and can be made much smaller than the frequency of external signal. These smaller oscillations which are called Rabi oscillations can be detected with the aid of a high quality tank circuit coupled inductively to the qubit loop.

Below we find the expression for the current in a qubit loop in presence of electromagnetic resonant excitation for zero and non zero temperatures. We show that for typical parameters of three-junction phase qubit the current oscillations in a qubit loop give rise to the voltage oscillations across the tank with the amplitude which can be at the μV level that makes a direct detection of Rabi oscillation possible.

II. QUANTUM DYNAMICS OF 3JJ PHASE QUBIT

Quantum dynamics of three-junction phase qubit has been studied in detail in Ref. 2. The qubit consists of a loop with three Josephson junctions. The loop has very small inductance, typically in pH range. It insures effective decoupling of qubit from external environment. Two Josephson junctions have equal critical current I_C and capacitance C, while the critical current and capacitance of a third junction is a little bit smaller, αI_C, αC where $0.5 < \alpha < 1$. If the Josephson coupling energy $E_J = I_C\Phi_0/2\pi$, where $\Phi_0 = h/2e$ is a flux quantum, is much more than the Coulomb energy $E_C = e^2/2C$, then the phase of a Cooper pair wave function is well defined. As was shown in Refs. 1,2 in the vicinity of $\Phi = \Phi_0/2$ this system has two quantum stable states which macroscopically differ by the direction of the current circulating in a qubit loop. In the absence of high frequency excitation the quantum properties of the qubit are described by Hamiltonian (Eq. (12) in Ref. 2):

$$H_0 = \frac{P_\varphi^2}{2M_\varphi} + \frac{P_\theta^2}{2M_\theta} + U(f,\phi,\theta) \tag{2}$$

where $P_\varphi = -i\hbar\partial/\partial\varphi$, $P_\theta = -i\hbar\partial/\partial\theta$, $M_\varphi = (\Phi_0/2\pi)^2 2C$, $M_\theta = (\Phi_0/2\pi)^2 2C(1+2\alpha)$;

$$\begin{aligned} U(f,\varphi,\theta) = E_J \{2 + \alpha - 2\cos\varphi\cos\theta \\ -\alpha\cos\left[2\pi\left(f+\frac{1}{2}\right)+2\theta\right]\right\} \end{aligned} \tag{3}$$

Here $\varphi = (\varphi_1+\varphi_2)/2$, $\theta = (\varphi_1-\varphi_2)/2)$, where φ_1, φ_2 are gauge-invariant phases of two Josephson junctions with equal critical currents. In contrast to Ref. 2, in (3) we define the flux bias $f = \Phi/\Phi_0 - \frac{1}{2}$ as a small parameter measuring the departure from degeneracy. At the degeneracy point $f = 0$ potential energy (3) shows two minima with equal energies ε_0 at the points $\varphi = 0$, $\theta = \pm\theta_c$, where $\cos\theta_c = 1/2\alpha$. The tunneling between the minima lifts the degeneracy leading to the energy levels $E_\pm = \varepsilon_0 \pm \Delta$ where Δ is a tunneling matrix element between two minima. However, at the degeneracy point the current in a qubit loop vanishes, so that it is necessary to move a little bit away from this point. In order to find the levels in the close vicinity of degeneracy point we expand potential energy (3) near its minima taking account for linear terms in f and quadratic terms in quantum variables ϕ, θ. With the use of the technique described in Ref. 2 we find the following expression for the energies of two levels:

$$E_\pm = \varepsilon_0 \pm \sqrt{E_J^2 f^2 \lambda^2(\alpha) + \Delta^2} \tag{4}$$

where we take offset at the degeneracy point, i. e., *f=0* corresponds to $\Phi_X = \Phi_0/2$;

$$\varepsilon_0 = E_J\left(\left(2-\frac{1}{2\alpha}\right) + \sqrt{\frac{1}{\alpha(E_J/E_C)}}\left(1+\sqrt{2\alpha-1}\right)\right) \tag{5}$$

$$\begin{aligned} \lambda(\alpha) = \ & \frac{\pi}{\alpha}\left(-\sqrt{4\alpha^2-1} + \sqrt{\frac{\alpha}{(E_J/E_C)}}\right. \\ & \left.\times\left(\frac{2\alpha^2-1}{\sqrt{4\alpha^2-1}} + \frac{2\alpha^2+1}{\sqrt{2\alpha+1}(4\alpha^2-1)}\right)\right) \end{aligned} \tag{6}$$

Expression (4) differs from corresponding equation in Ref. 11 by a factor $\lambda(\alpha)$ which explicitly accounts for the dependence of the energies $E_\pm$ on α and E_J/E_C. The stationary state wave functions $\Psi_\pm$ are eigenfunctions of Hamiltonian H_0 : $H_0\Psi_\pm = E_\pm\Psi_\pm$. They can be written as the superpositions of the wave functions in the flux basis, Ψ_L, Ψ_R where L, R stand for the left, right well, respectively: $\Psi_\pm = a_\pm\Psi_L + b_\pm\Psi_R$.

$$a_\pm = \frac{\Delta}{\sqrt{(\varepsilon_+ - E_\pm)^2 + \Delta^2}}; b_\pm = \frac{\varepsilon_+ - E_\pm}{\sqrt{(\varepsilon_+ - E_\pm)^2 + \Delta^2}}; \tag{7}$$

where $\varepsilon_+ = \langle\Psi_L|H_0|\Psi_L\rangle$; $\varepsilon_- = \langle\Psi_R|H_0|\Psi_R\rangle$. For stationary states the current circulating in a qubit loop can be calculated either as the average of a current operator $\widehat{I_q} = I_C\sin(\varphi+\theta)$ over stationary wave functions or as a derivative of the energy over the external flux:

$$I_q = \langle\Psi_\pm|\widehat{I_q}|\Psi_\pm\rangle = \frac{\partial E_\pm}{\partial\Phi} = \pm I_C f \frac{\lambda^2(\alpha)}{\pi}\frac{E_J}{\hbar\omega_0} \tag{8}$$

where $\hbar\omega_0 = E_+ - E_-$.

Suppose we apply to the qubit the excitation on a frequency close to a gap frequency $\omega = \hbar\omega_0 \approx 1$ GHz. The corresponding perturbation term is then added to Hamiltonian H_0: $H_{int} = V(\phi,\theta)\cos(\omega t)$ where $V(\phi,\theta)$ in the vicinity of the left (right) minimum is as follows:

$$\begin{aligned} V_{L,R}(\theta,\varphi) = \ & E_J\frac{\pi}{\alpha}f_{ac}\left(\mp\sqrt{4\alpha^2-1} \pm \varphi^2\frac{2\alpha^2-1}{\sqrt{4\alpha^2-1}}\right. \\ & \left.\pm(\theta-\theta_C^\pm)^2\frac{2\alpha^2+1}{\sqrt{4\alpha^2-1}}\right) \end{aligned} \tag{9}$$

$$\theta_C^\pm = \pm\theta_C + 2\pi f\frac{1-2\alpha^2}{4\alpha^2-1} \tag{10}$$

Throughout the paper we assume $f \geq 0$, so that the bottom of the left well is higher than the bottom of the right well. Accordingly, in Eqs. (9) and (10) the upper sign refers to right minimum, the lower sign refers to left minimum. The quantity f_{ac} in Eq. (9) is the amplitude of excitation field in flux units: $f_{ac} = \Phi_{ac}/\Phi_0$. The high frequency excitation induces the transitions between two levels which result in a superposition state for the wave function of the system: $\Psi(t) = C_+(t)\Psi_+ + C_-(t)\Psi_-$. The coefficients $C_\pm(t)$ are obtained from the solution of time dependent Schrodinger equation with proper initial conditions for $C_\pm(t)$. We assume that before the excitation the system was at the lower energy level: $C_-(t=0) = 1; C_+(t=0) = 0$. The corresponding solution for $C_\pm(t)$ in the rotating wave approximation is as

follows:[13]

$$C_-(t) = e^{-i\frac{E_-}{\hbar}t}e^{i\frac{\omega-\omega_0}{2}t}\left[\cos\Omega t - i\frac{\omega-\omega_0}{2\Omega}\sin\Omega t\right]$$

$$C_+(t) = e^{-i\frac{E_+}{\hbar}t}e^{-i\frac{\omega-\omega_0}{2}t}\left(-i\frac{\Omega_r}{\Omega}\right)\sin\Omega t \quad (11)$$

where $\Omega_r = \frac{|\langle\Psi_+|V|\Psi_-\rangle}{2\hbar}$, $\Omega = \sqrt{\frac{(\omega-\omega_0)^2}{4} + \Omega_r^2}$. At resonance ($\omega = \omega_0$) we get $\Omega = \Omega_r$. Taking Ψ_L, Ψ_R as ground state oscillatory wave functions in the left, right well, respectively, we calculate matrix element $\langle\Psi_+|V|\Psi_-\rangle$ and obtain for Rabi frequency:

$$\Omega_r = \frac{E_J}{\hbar} f_{ac}|\lambda(\alpha)|\frac{\Delta}{\hbar\omega_0} \quad (12)$$

As is seen from Eq. (12) by a proper choice of excitation power (f_{ac} in our case) the frequency Ω_r can be made much lower the gap frequency Δ/h. Now we calculate the average current in a superposition state:

$$I_q = \langle\Psi(t)|\widehat{I_q}|\Psi(t)\rangle$$
$$= |C_+(t)|^2\langle\Psi_+|\widehat{I_q}|\Psi_+\rangle + |C_-(t)|^2\langle\Psi_-|\widehat{I_q}|\Psi_-\rangle \quad (13)$$

where we neglect high frequency ω_0 term. Accounting for the expression (8) for the current in stationary states we obtain at resonance the following expression for the average current flowing in a qubit loop:

$$I_q = -I_C f\frac{\lambda^2(\alpha)}{\pi}\frac{E_J}{\hbar\omega_0}\cos 2\Omega_r t \quad (14)$$

At finite temperature the calculation of the current in a qubit loop in presence of high frequency excitation is based on the density matrix equation: $i\hbar\dot{\rho}(t) = [(H_0 + H_{int}(t)), \rho(t)]$ with initial conditions at thermal equilibrium: $\rho_{++}(0) = \rho_{++}^{eq}$; $\rho_{--}(0) = \rho_{--}^{eq}$; $\rho_{+-}(0) = \rho_{-+}(0) = 0$, where ρ_{++}^{eq}, ρ_{--}^{eq} are equilibrium density matrix elements : $\rho_{++}^{eq} = \frac{1}{Z}e^{-\frac{E_+}{k_BT}}$; $\rho_{--}^{eq} = \frac{1}{Z}e^{-\frac{E_-}{k_BT}}$, $Z = e^{-\frac{E_+}{k_BT}} + e^{-\frac{E_-}{k_BT}}$.

In the rotating wave approximation the diagonal elements of density matrix at resonance ($\omega = \omega_0$) are as follows:

$$\rho_{--}(t) = \frac{1}{2} + \frac{1}{2}\tanh\frac{\hbar\omega_0}{2k_bT}\cos 2\Omega_r t, \quad (15)$$

$\rho_{++}(t) = 1 - \rho_{--}(t)$.

Now we find the average current at resonance for nonzero temperature:

$$I_q = \langle\Psi_+|\widehat{I_q}|\Psi_+\rangle\rho_{++}(t) + \langle\Psi_-|\widehat{I_q}|\Psi_-\rangle\rho_{--}(t)$$
$$= -I_C\frac{E_J f\lambda^2(\alpha)}{\pi\hbar\omega_0}\tanh\left(\frac{\hbar\omega_0}{2k_BT}\right)\cos 2\Omega_r t \quad (16)$$

In order to estimate the Rabi frequency Ω_r we take the following qubit parameters: $I_C = 400$ nA, $\Delta/h = 0.3$ GHz, $\alpha = 0.8$, $L = 15$ pH, $E_J/E_C = 100$. We take the amplitude of time-dependent flux which is coupled to qubit from high frequency source $f_{ac} = 1\times10^{-4}$. We set the flux offset from degeneracy point $f = 3.5\times10^{-4}$, so that $\hbar\omega_0 = 2\sqrt{2}\Delta$, $\omega_0/2\pi = 0.85$ GHz. For these values we obtain from Eq. (12)the Rabi frequency $\Omega_r/2\pi = 32$ MHz.

III. THE INTERACTION OF THE PHASE QUBIT WITH A TANK CIRCUIT

The problem of a coupling a quantum object to the classical one, which is dissipative in its nature, has no unique theoretical solution. A rigorous approach is to start from exact Hamiltonian which describes the qubit-tank circuit system:

$$H = H_0 + H_T + H_{0T} + H_{TB} + H_B \quad (17)$$

where H_0 is the qubit Hamiltonian given in Eq. (2), H_T is the tank circuit Hamiltonian

$$H_T = \frac{Q^2}{2C_T} + \frac{\Phi^2}{2L_T} \quad (18)$$

where Q and Φ are a quantum operators of the charge at the capacitor and of magnetic flux trapped by the inductor of a tank circuit, respectively. The operators obey commutator relation $[\Phi, Q] = i\hbar$. The interaction Hamiltonian between the qubit and the tank, H_{0T} is:

$$H_{0T} = \frac{M}{L}\hat{I}_q\Phi \quad (19)$$

where M is inductive coupling between qubit and the tank; H_B is the Hamiltonian of a thermal bath coupled to the tank via interaction $H_{TB} = \alpha\Phi\Gamma$, where α is the coupling constant between the tank and dissipative environment, Γ is the dynamic variable of thermal bath H_B.

The equations of motion for tank circuit variables are as follows:

$$\frac{dQ}{dt} = -\frac{\Phi}{L_T} - \frac{M}{L}\hat{I}_q + \alpha\Gamma \quad (20)$$

$$\frac{d\Phi}{dt} = \frac{Q}{C_T} \quad (21)$$

From these two equations we get for the voltage operator $\hat{V} = Q/C_T$ across the tank:

$$\frac{d^2\hat{V}}{dt^2} + \omega_T^2\hat{V} = -M\omega_T^2\frac{d\hat{I}_q}{dt} + \alpha\frac{d\Gamma}{dt} \quad (22)$$

The averaging of this equation over the bath leads to the dissipative equation for the average voltage, V, across the tank (see, for example, Ref. 14):

$$\ddot{V} + \frac{\omega_T}{Q_T}\dot{V} + \omega_T^2 V = -M\omega_T^2\frac{dI_q}{dt}. \quad (23)$$

where $Q_T = \omega_T R_T C_T \gg 1$ is the tank quality factor, $\omega_T = 1/\sqrt{L_T C_T}$.

In the spirit of selective quantum evolution approach,[8] we interpret Eq. 23 as follows. Suppose the qubit is in a pure state, $a|0\rangle + b|1\rangle$. The tank voltage is measured (in quantum-mechanical sense) at certain times $t_k = \Delta t, 2\Delta t, ...$ Each time the qubit state is also measured, since there is correspondence between $V(t_k)$ and qubit current, which thereby takes value either $\langle 0|\hat{I}_q(t_k)|0\rangle$ or $\langle 1|\hat{I}_q(t_k)|1\rangle$, with appropriate probability. Averaging $V(t_k)$ over intervals $\Delta T \gg \Delta t$,which are still small compared to other characteristic times in the system, would yield Eq. 23.

The quantity to be detected is the oscillating voltage across the tank which at resonance is $V = V_a \cos 2\Omega_r t$, where the voltage amplitude, V_a is:

$$V_a = MQI_C f \frac{2\lambda^2(\alpha)}{\pi} \frac{E_J}{\hbar\omega_0} \Omega_r. \tag{24}$$

It is the presence of these voltage oscillations in the detected signal which reveals the existence of Rabi oscillations in the qubit.

The Eq. (23) is still quantum equation since the average voltage V is a quantum operator of the voltage across the tank averaged over the bath degrees of freedom. Therefore, our problem is reduced to the problem of measuring a weak external force (in our case, MdI_q/dt) by a dissipative oscillator. As is known (see, for example, Ref. 15) a classical descriptions of such oscillator requires that the quantum fluctuations of the detector variable (i. e. V in our case) in the measurement bandwidth be smaller than the amplitude induced in the tank coil by external signal, $MQ_T dI_q/dt$. According to the fluctuation-dissipation theorem the quantum fluctuations of the voltage V are given by the spectral density:

$$S_V(\omega) = 2ReZ(\omega)\hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right) \tag{25}$$

where $Z^{-1}(\omega) = (1/R_T + 1/i\omega L_T + i\omega C_T)$ is the impedance of a tank circuit.

Below we take the following parameters of the tank circuit: $L_T = 50$ nH, $Q = 1000$, inductive coupling to qubit, $k^2 = M^2/LL_T = 10^{-4}$. We assume the tank is tuned to the frequency of oscillating current in the qubit, i. e., $\omega_T/2\pi = 2\Omega_r/2\pi = 64$ MHz, hence, $C_T = 124$ pF. We take the flux amplitude which is coupled to qubit from high frequency source $f_{ac} = 1 \times 10^{-4}$. We set the flux offset from degeneracy point $f = 3.5 \times 10^{-4}$, so that $\hbar\omega_0 = 2\sqrt{2}\Delta$, $\omega_0/2\pi = 0.85$ GHz. For these values we obtain for the voltage amplitude at resonance $V_a \approx 0.7$ μV.

From Eq. (25) we estimate the voltage fluctuations across the tank circuit coil at resonance ($\omega = \omega_T$) :

$$V_n = \sqrt{\frac{S_V(\omega_T)}{Z(\omega_T)}} \omega_T L_T Q_T \sqrt{B} \tag{26}$$

where $B = \omega_T/2\pi Q_T$ is the bandwidth of the tank circuit. We perform the calculations for $T = 10$ mK. For the voltage fluctuations we get $V_n \approx 10$ nV. Thus, we see that the voltage fluctuations across the tank coil is much smaller than the signal amplitude from the qubit. Therefore, we may treat the tank circuit as a classical object and the voltage $V(t)$ as the classical variable coupled through Eq. 23 to the qubit degree of freedom, where the current I_q in Eq. 23 is calculated as the average of a current quantum operator over the qubit statistical operator.

IV. THE EFFECTS OF QUBIT RELAXATION AND DECOHERENCE

The estimations we made above are very promising, however, our derivation is made under a strong assumption: we neglected the decoherence due to interaction of the qubit with external environment and with a measuring device. In fact, the possibility of detection of Rabi frequency depends crucially on relaxation, Γ_r, and decoherence, Γ_φ rates, which lead to the decay of Rabi oscillations. The detection is in principle possible if period of oscillations is small compared to $min[1/\Gamma_\varphi, 1/\Gamma_r]$. The decoherence is caused primarily by coupling of a solid state based phase qubit to microscopic degrees of freedom in the solid. Fortunately this intrinsic decoherence has been found to be quite weak:[16] the intrinsic decoherence times appeared to be on the order of 1 ms which is several orders of magnitude more than period of current oscillations we estimated before: $\pi/\Omega_r = 15.6$ ns. However, the external sources of decoherence are more serious. In our method these are the microwave source which induces the Rabi oscillations and the tank circuit which has to detect them. From this point two structures of microwave source has been analyzed: coaxial line that is inductively coupled to the qubit[17] and on-chip oscillator based on overdamped DC SQUID.[18] The analysis has shown the relaxation and decoherence rates were on the order of 100 μs at 30 mK for coaxial line, and 150 μs and 300 μs for relaxation and decoherence, respectively, for on-chip oscillator at 1 GHz. Here we estimate decoherence times that are due to a tank circuit using the expressions for Γ_r and Γ_φ from[19]

$$\Gamma_r \equiv \frac{1}{T_r} = \frac{1}{2}\left(\frac{\Delta}{\hbar\omega_0}\right)^2 J(\omega_0) \coth\left(\frac{\hbar\omega_0}{2k_B T}\right) \tag{27}$$

$$\Gamma_\varphi \equiv \frac{1}{T_\varphi} = \frac{\Gamma_r}{2} + 2\pi\eta\left(\frac{E_J f\lambda}{\hbar\omega_0}\right)^2 \frac{k_B T}{\hbar} \tag{28}$$

where dimensionless parameter η reflects the Ohmic dissipation. It depends on the strength of noise coupling to the qubit. In our subsequent estimations we take $\eta = 5 \times 10^{-3}$ which is relevant for weak damping limit. Below we use the approach described in Ref. 20 in its simplified form.[17,18,21] While it is not mathematically rigorous, nevertheless it gives a correct order of magnitude of relaxation times. The quantity $J(\omega_0)$ is the zero

temperature spectral density of the fluctuations of the gap energy of the qubit: $J(\omega) =< \delta\varepsilon(\omega)\delta\varepsilon(\omega) > /\hbar^2$. The fluctuations $\delta\varepsilon$ are due to the flux noise δf, which is supplied to the qubit by a tank circuit. From Eq.(5) we get $\delta\varepsilon = \left(4E_J^2\lambda^2 f/\hbar\omega_0\right)\delta f$, where $\delta f = M\delta I/\Phi_0$. The current noise δI in a tank circuit inductance comes from two independent parts: Johnson-Nyquist voltage noise in a tank circuit resistance with a spectral density at $T = 0$: $S_V(\omega) = 2\hbar\omega ReZ(\omega)$, where $Z^{-1}(\omega) = (1/R_T + 1/i\omega L_T + i\omega C_T)$ is the impedance of a tank circuit, and from a current noise of preamplifier with a spectral density S_A. Therefore, we get for $J(\omega_0)$: $J(\omega_0) = \left(4E_J^2\lambda^2 fM/\hbar^2\omega_0\Phi_0\right)^2 S_I(\omega_0)$, where $S_I(\omega) = \left[S_V(\omega)ReZ(\omega) + S_A\left|Z(\omega)\right|^2\right]/\omega^2L^2$ is the spectral density of a current noise in the tank circuit inductance. Since $\omega_0 >> \omega_T, S_I(\omega_0) \cong (2\hbar\omega_0/R_T + S_A)\left(\omega_T/\omega_0\right)^4$. For the estimation we take $T = 10$ mK, $R_T = 20$ kΩ, $S_A = 10^{-26}$ A^2/Hz, $\omega_T/2\pi = 64$ MHz, other parameters being the same as before. We find that the contribution to the relaxation of a tank circuit noise and of preamplifier noise is approximately 0.1 s^{-1} and 22 s^{-1}, respectively. Therefore total relaxation is determined by preamplifier noise giving relaxation time $T_r \cong 45$ ms. The decoherence rate is dominated by a second term in (28), which is equal 5.4×10^6 s^{-1} giving decoherence time $T_\varphi = 185$ ns, which is approximately ten oscillation periods of circulating current. These estimations clearly show the possibility of detection of Rabi frequency in MHz range with the aid of conventional rf circuitry.

Here we did not consider the effect of a tank circuit back action on the qubit which can give additional contribution to the relaxation and decoherence rates. In order to reduce the back action effect of a tank circuit and to further enhance the MQC signal it may be advantageous for detection of Rabi oscillations to use a two dimensional array of identical phase qubits coupled to a tank circuit. Modern technology allows several thousands of weakly coupled phase qubits to be obtained on a chip.[17] Preliminary experiments showed that the behavior of macroscopic current in such a system is completely analogous to that of longitudinal magnetization in NMR: the collective reversal of the persistent currents in the qubit loops when sweeping the flux bias within a degeneracy point has been observed.[17]

In conclusion, we have shown that Rabi oscillations (in MHz range) of the current circulating in a qubit loop which are induced by high frequency external source (in GHz range) can be detected in MHz range as the voltage oscillations in the high quality tuned tank circuit inductively coupled to the qubit.

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